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Prediction of Crack Propagation Using Finite Element Method

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ABSTRACT
Cracks occur in many structural parts due to various causes. Stress concentrations at the crack tips and crack propagation due to tensile stresses are active areas of research in the past many decades. Finite element analysis of prediction of crack growth has been cited in many the literature. Element distortions, breakage of elements, coarse mesh, lower order elements, material and geometric non-linearity etc are some of the issues faced in the analysis of cracks with finite elements. An attempt towards improving the prediction of crack propagation using finely refined mesh near the crack tip, higher order elements and better material models as available in the ANSYS software is the aim of this thesis. A rectangular plate with a crack originating due to impact loading and propagating towards one direction is taken as an example to illustrate this.

***Keywords* -** Crack opening displacement, Crack tip velocity, Fracture mechanics, Fracture toughness, Stress intensity factor

I. Introduction

Existence and propagation of cracks in steel and concrete elements in civil engineering structures is quite important since it affects very much the ultimate mechanical strength and resistance of the structure to environmental effects. The study of the crack propagation is also quite important to estimate the ultimate strength and the failure procedures are such structures especially during earthquakes. Failure of the engineering structures is caused by cracks. Cracks are present to some extent in all structures, either as a result of manufacturing defects or localized damage during service. The crack growth leads to a decrease in the structural strength. Thus, cracks lead to failure of the structure during service loading. Fracture, the final catastrophic event takes place very rapidly and is preceded by crack growth, which develops slowly during normal service conditions.

Fracture mechanics has developed into a useful discipline for predicting strength and life of cracked structures. Linear elastic fracture mechanics can be used in damage tolerance analysis to describe the behavior of a crack. The fundamental assumption of linear elastic fracture mechanics is that the crack behavior is determined solely by the values of the stress intensity factors which are functions of the applied load and the geometry of the cracked structure. The stress intensity factors thus play a fundamental role in linear elastic fracture mechanics applications. Fracture mechanics deals with the study of how a crack in a structure propagates under applied loads. Fracture parameters such as stress intensity factors which are used to estimate the crack growth are indicators of analytical prediction of crack propagation.

Shahani and Fasakhodi [1] presents a finite element analysis based on the remeshing technique to predict the dynamic crack propagation and crack arrest in a brittle material, namely Araldite-B. T Nishioka and Atluri [2] done numerical analysis of dynamic crack propagation in Araldite B material under plane stress condition. Alshamma and Fahem [3] study the effect of impact loading on dynamic crack propagation in thin and isotropic thick plates for two types of material, stainless steel and aluminum, by analytically and numerically. Weisbrod and Rittel [4] studied the dynamic fracture toughness testing of small beam specimens and calculate sif experimentally. Kishimoto *et. al* [5] analyze the time history of dynamic sif for a dynamic one point bend test with an edge cracked specimen impacted at mid span without support.

In the present work, the numerical investigation of the crack propagation in a rectangular plate with a crack originating due to impact loading is to be carried out through a Finite element analysis of the two-dimensional (rectangular) domain. Element distortions, breakage of elements, material and geometric non-linearity, etc. are some of the issues faced in the analysis of cracks with the finite element method. An attempt towards improving the prediction of crack propagation using finely refined mesh near the crack tip, higher order elements and better material models as available in the ANSYS software is used to perform the work.

II. One Point Impact Experiment

Inorder to determine the dynamic stress intensity factor of a commercial tungsten base heavy alloy, Weisbrod and Rittel(2000), done a one point impact experiments on a short Charpy beam specimen subjected to impact loading. An overview of the method, including the experimental setup, is shown in Fig 2.1.

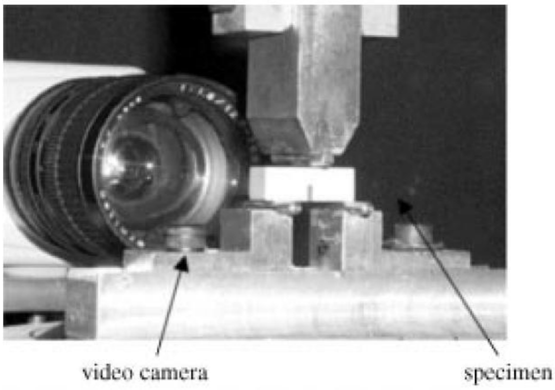

Fig 2.1: Experimental setup

Dynamic loads are applied using a single instrumented bar, to perform one point impact experiments. The apparatus consists of a cylindrical incident bar instrumented at its mid-length with a pair of diametrically cemented strain gages. Stress wave loading is applied by means of an air propelled cylindrical striker. The length of the striker and its velocity at impact set the duration and amplitude of the stress wave which loads the fracture specimen. Both the incident bar and the striker are made of a commercial tungsten base heavy alloy (w/o-90W-7Ni-3Fe). The specimen lays simply supported, and is in contact with the bar. Consequently, fracture results from inertia only, as typical of one point bend (1PB) impact fracture.

The experimental specimens are of the short Charpy type, whose geometry and dimensions shown in Fig 2.2. All the specimens were fatigue pre cracked on a servo-hydraulic machine (MTS-810), according to ASTM standard recommendations (ASTM-E399, 1993). Crack growth monitoring was carried out by means of two video cameras. The experimental material was a commercial tungsten base heavy alloy (w/o-90W-7Ni-3Fe), whose modulus of elasticity is 338 GPa and Poissons ratio 0.3. The density was found to be 17100 kg/m^3. Poisson's ratio was determined for two orthogonal specimen orientations with respect to the applied load. These measurements showed very little difference between the two directions so that the material can reasonably be considered as isotropic for computational purposes. Young's modulus was determined from the measured longitudinal wave velocity in the incident bar. This value represents the dynamic Young's modulus which is used in numerical calculations to accurately reproduce wave propagation in the specimen.

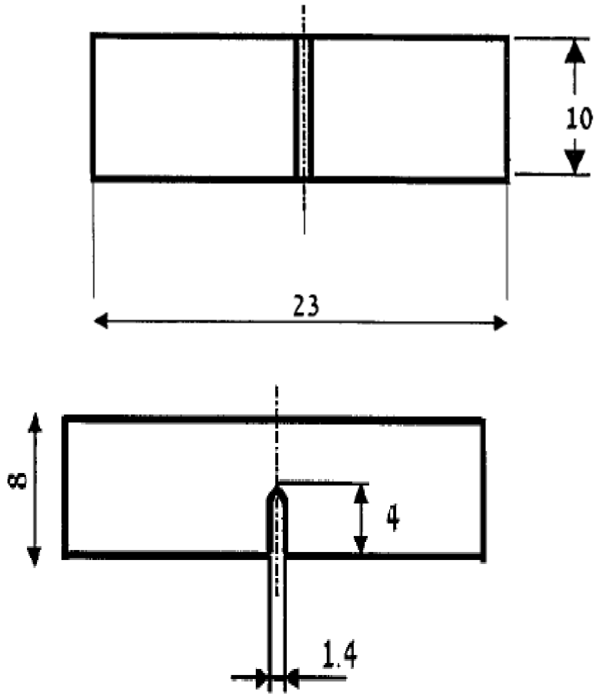

Fig 2.2: The Short Charpy specimen(All dimensions are in mm).

III. Numerical Experiment of Crack Growth on a Charpy Impact Test Specimen

Here the 2D model of Charpy beam specimen is modeled and analyses by using a well known finite element software ANSYS. Due to symmetry considerations, a two dimensional representation of half of the specimen was adopted. Plane strain conditions were assumed, and the material was modeled as linear-elastic. The crack-tip singularity was enforced by quarter point six-node triangular isoparametric element. The loading provided is of 15 kN maximum load in 40 micro second duration. The discretized specimen is shown in Fig 3.1.

The full transient dynamic analysis is performed on the specimen along with inertia effects and variations of crack tip opening displacement with time is plotted. The graph is somewhat similar to the graph which is obtained from the experimental result by Rittel and Weisbrod [4]. The same specimen is again remeshed by increasing the number of elements to double that of the initial one and the same graph is plotted as in Fig 3.2. The result shows that decreasing the size of the element in a finite element model increases the accuracy of result to exact values.

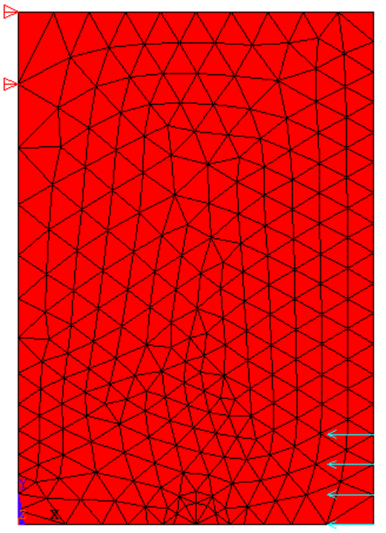

Fig 3.1: Discretized 2D half crack model with loading(n = 986)

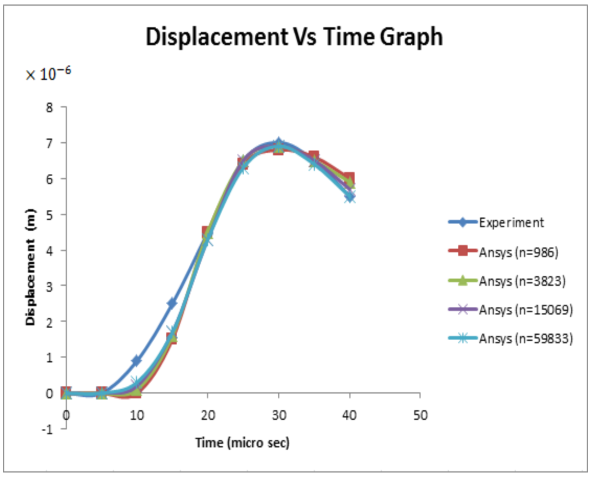

Fig 3.2: Comparison of Displacement Vs Time graphs for various 2D Models

3.1 Determination of Stress Intensity Factor:

The Stress intensity Factor was calculated from Irwin's formula which relate crack opening displacement to the SIF [6]. The expression for plane strain is :

$$COD(t) = KI\ (t) \times \frac{8\ (1-\nu^2)}{E} \times \sqrt{\frac{r}{2\pi}} \qquad (3.1)$$

Defining V(t) = COD(t) / 2, due to symmetry,

$$\mathrm{KI}(t) = \frac{V(t) \times E}{4 \times (1-\vartheta^2)} \times \sqrt{\frac{2\pi}{r}} \qquad (3.2)$$

where V(t) is the displacement of a selected point located at a distance r = 0.5 mm from the crack tip[7]. From the above expressions, the SIF can be calculated and plotted on a graph against time. This graph is also similar to that obtained from journal as shown in Fig 3.3.

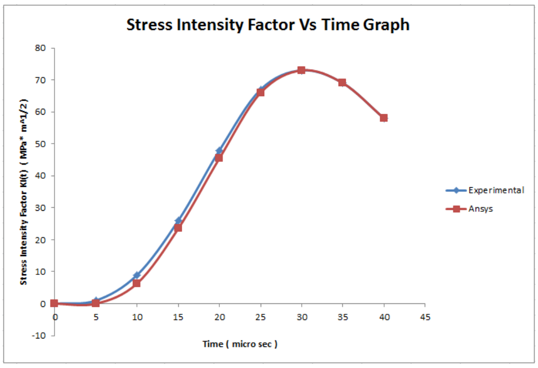

Fig 3.3: Validation of SIF Vs Time graph for 2D Models

3.2 Crack Tip Velocity

Freund performed a more detailed numerical analysis for a dynamically propagating crack in a finite body and obtained the following relationship [6].

$$V = c_r \times \left[1 - \frac{a_0}{a}\right] \qquad (3.3)$$

where c_r is the Raleigh(surface) wave speed.

For Poisson's ratio = 0.3, the $\frac{c_r}{c_0}$ ratio= 0.57 [6]

where c_o is the speed of sound in one dimensional wave propagation.

a_o is the initial crack length

a is the original crack length

From the above expression crack tip velocity can be calculated and a graph is plotted between crack tip velocity Vs time as shown in the Fig 3.4.

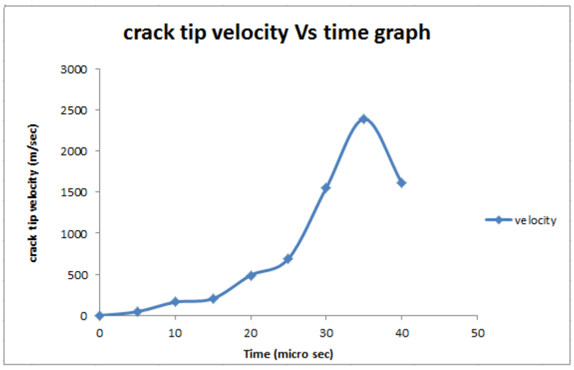

Fig 3.4: crack tip velocity Vs Time graph

3.3 Dynamic Stress Intensity Factor

The governing equation for Mode I crack propagation under elasto dynamic condition can be written as

$K_I\ (t) = K_{ID}(V)$, where K_I is the instantaneous stress intensity factor and K_{ID} is the material resistance to crack propagation, which depends on the crack velocity [8].

In general $K_I\ (t)$ is not equal to the static stress intensity factor as defined earlier. A number of researchers have obtained a relationship for the dynamic stress intensity factor of the form,

$$K_I(t) = k(V) \times K_I(0) \qquad (3.4)$$

where $k(V)$ is the universal function of crack speed $K_I(0)$ is the static stress intensity factor.
The function $k(V) = 1$, when $V=0$, and decreases to zero as V approaches the Rayleigh wave velocity.
The approximate expression for k is represented as,

$$k(V) \approx \left[1 - \frac{V}{c_r}\right] \times \sqrt{(1 - hV)} \qquad (3.5)$$

where 'h' is the function of elastic wave speeds and can be approximated by,

$$h \approx \frac{2}{c_1} \times \left(\frac{c_2}{c_1}\right)^2 \times \left[1 - \left(\frac{c_2}{c_1}\right)\right]^2 \qquad (3.6)$$

where 'c_1' and 'c_2' are the longitudinal and shear wave speeds respectively.

$c_1 = \sqrt{\left(\frac{E}{\rho}\right)}$ where E is the Young's modulus of tungsten alloy, and

$c_2 = \sqrt{\left(\frac{G}{\rho}\right)}$, where G is the Shear modulus of tungsten alloy.

The calculated dynamic stress intensity factor is then plotted against time and obtained a graph as shown in Fig 3.5.

3.4 Dynamic Fracture Toughness

During unstable crack propagation, the fracture toughness of the material is introduced as dynamic fracture toughness, and is denoted by K_{IC}^d which is not a constant value. The quantity K_{IC}^d represents the resistance of the material to crack growth. The magnitude of K_{IC}^d in a special temperature is expected to depend on the crack speed and on the properties of the material. All inertial, plasticity and rate effects are lumped into the material property K_{IC}^d [6]. Several experiments indicate that the materials level of resistance to crack advance may depend on the instantaneous crack tip speed. The most significant feature of the speed dependency is the increasing sensitivity of dynamic fracture toughness to crack tip speed with increasing speed.
The dynamic fracture toughness and crack tip velocity can be correlated by the following equation:

$$K_{IC}^d = \frac{K_{IA}}{1 - \left(\frac{V}{V_l}\right)^m} \qquad (3.7)$$

Where K_{IA} and 'm' are material constants that must be determined experimentally,
V_l Corresponds to a limiting crack speed, K_{IA} corresponds to toughness of material.

The Fig 3.6 shows dynamic fracture toughness Vs. crack tip speed for tungsten alloy specimen.

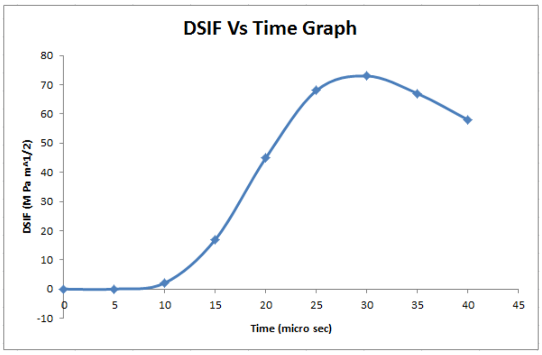

Fig 3.5: Dynamic stress intensity factor Vs Time Graph

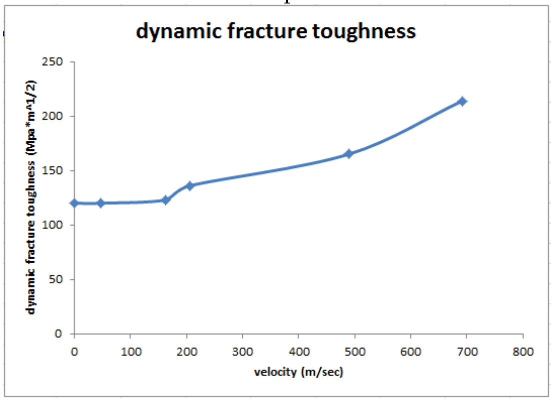

Fig 3.6: dynamic fracture toughness Vs crack tip velocity

3.5 Measurement of Errors

Error estimates helps to give an idea of the accuracy of the approximate solution. When finite element computations are performed on a computer, round off errors introduced into the solution. As we refine the mesh, the domain is more accurately represented and therefore, the boundary approximation errors are expected to zero [9].

One of the main objective of this thesis is to find the error ($E=u-u^a$) between finite element solution and exact solution, also to provide possible solutions to bring the finite element solution close to exact results. Where 'u' is the exact solution and u^a is the finite element solution. Therefore, the error in the approximation can be reduced either by reducing the size of elements or increasing the degree of approximation. Convergence of finite element solutions with mesh refinements (i.e more of the same kind of elements are used) is termed h-convergence. Convergence with increasing degree of polynomials is called p-convergence.

There are several ways in which we can measure difference (or distance) between any two functions u and u^a .The point wise error is the difference of u and u^a at each point of domain. More generally used measures or (norms) of the difference of two functions are the L^2 error norm and energy norm. For any square integrable functions u and u^a

defined on the domain(a, b) ,the two norms are defined by,

$$\text{Relative } L_2 \text{ error norm} = \frac{\|U-U_a\|}{\|U\|} \qquad (3.8)$$

$$\|U-U_a\| = \sum_{i=1}^{n}\sqrt{(U_i - U_i^a)^2} \qquad (3.9)$$

$$\|U\| = \sum_{i=1}^{n}\sqrt{(U_i)^2} \qquad (3.10)$$

Where ,

u = exact solution (Experimental value)

u^a = Finite element solution

From the displacements of various meshes we can calculate relative L^2 error norm for each model[9].

Therefore, Relative L^2 error norm = 1.456/13.8224
= 0.1053 (For n = 986)

Relative L^2 error norm = 1.2728/13.8224
= 0.0921 (For n = 3823)

Relative L^2 error norm = 1.1045/13.8224
= 0.0799 (For n = 15069)

Relative L^2 error norm = 0.956/13.8224
= 0.0692 (For n = 59833)

From the above results, a graph is plotted between Relative L^2 error norm Vs (no. of nodes) in log scale and the graph obtained is as shown in the Fig 3.7.

The log log plots give the rates of convergence in the norms. The rates of covergence are given by the slope of line.

∴ Rate of convergence = 0.104

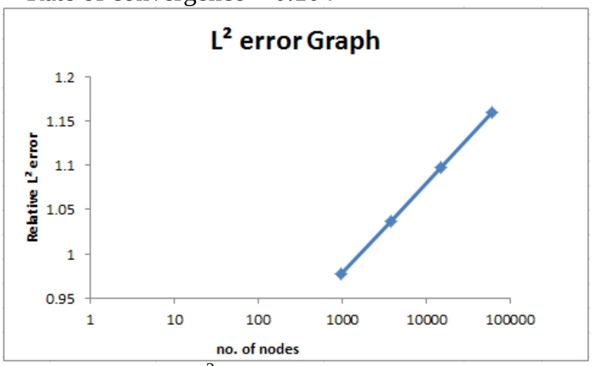

Fig 3.7: Relative L^2 error norm Vs no. of nodes

IV. CONCLUSION

The problem of dynamic crack propagation in Tungsten based heavy alloy specimen, has been analyzed. Finite element analysis is used to simulate the crack growth during dynamic crack propagation. The crack tip opening displacement is calculated from the analysis and it is plotted against time. The graph shows good agreement with the experimental study conducted on a short Charpy specimen. From the displacements, other parameters such as dynamic stress intensity factor, crack tip velocity, dynamic fracture toughness etc. can be calculated using the numerical equations.

The study also focuses on the error reduction methods in finite element analysis. The main methods used are 'p' refinement and 'h' refinement. 'p' refinement concentrate on increasing the order of polynomials. 'h' refinement concentrate on increasing the no. of elements in the domain. This project concentrate on 'h' refinement and it is concluded that, by increasing the no. of elements in a finite element model we can increase the accuracy of the result.

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